

Local Criterion of Quality for Color Difference Formula

Sergey Bezryadin, KWE Int. Inc, San Francisco, USA; Pavel Bourov, UniqueIC's, Saratov, Russia.

Abstract

It is important to be equipped with criterion of correspondence between color difference formula and experimental data of color discrimination of standard observer considering a quality of color printing or measuring. Such criterion is used to distinguish whether applied color difference formula is good enough. A well-known criterion is the value of ellipticity of MacAdam ellipses. In this paper we introduce a criterion based on newer and more thorough experimental data obtained by Wyszecki & Fielder.

We compare some well known color difference formulas using proposed criterion and show that they are not optimal for metering of color printing and measuring quality. Also we state that CIE XYZ should not be thought as an orthonormal space.

Introducing a Cohen metric allows us to calculate a color difference more precise compared with CIE standard formulas (including CIE L*a*b* ΔE, ΔE 1994 and CIE DE 2000) with a great simplification of computation procedure.

Criterion of Local Non-Uniformity

Every time we talk about precision of color reproduction and there is a necessity to numerically evaluate a value of error, we have to calculate a color difference. Since it was introduced in 1976, color difference formula was modified several times. However, it is still not convenient enough from practical point of view. In this article we show, that XYZ and Lab systems are not the best choice for establishing a color difference formula. Most of colorimetric experiments allow a vector space approach for stimuli description, so we discuss several metrics based on distance definition traditional for vector algebra.

To evaluate metric quality we use criteria of local non-uniformity based on Wyszecki & Fielder experiments.

The shape of Wyszecki & Fielder ellipsoids can serve as a criterion of the chosen metric quality. From a practical point of view, the closer Wyszecki & Fielder ellipsoids' shape approximate the spheres with the same radius, the better the corresponding metric, the better it reflects the peculiarity of human color perception.

In this paper, the ratio of the largest distance to the smallest distance between the center of ellipse and its' surface was used as the criterion of imperfection of metric:

$$k_i = D_{\max,i} / D_{\min,i} \quad (1)$$

This criterion can be used not only for the estimate of the quality of the metric in color vector space, but also for an arbitrary metric constructed for any coordinate system, including CIE standards.

For each of ellipsoids, 10,000 points on its surface were chosen by Monte-Carlo method. After that, for considered coordinate system,

coordinates of each point and $D_{n,i}$, the distances from those points to the central point of the ellipsoid, were calculated based on the chosen metric. Then the largest and the smallest distances $D_{\max,i}$ and $D_{\min,i}$ were calculated in order to find their ratio k_i .

The maximum value of k_i and RMS (root mean square) were computed for estimate of the metric quality:

$$k_{\max} = \max(k_i) \quad (2)$$

where i can have values from 1 to 84 ;

$$k_{\text{RMS}} = \sqrt{\frac{1}{N} \sum_{i=1}^N k_i^2} \quad (3)$$

where $N = 84$.

Linear Color Coordinate System

If we accept a statement that any color stimulus might be represented by a vector in a linear vector space as a foundation, and use Euclid norm of a difference of two vectors as a color difference, then for an oblique coordinate system the color difference formula should be used as follows:

$$D^2 = g_{11} \cdot \Delta X^2 + g_{22} \cdot \Delta Y^2 + g_{33} \cdot \Delta Z^2 + 2g_{12} \cdot \Delta X \cdot \Delta Y + 2g_{13} \cdot \Delta X \cdot \Delta Z + 2g_{23} \cdot \Delta Y \cdot \Delta Z \quad (4)$$

Where the coefficients g_{ij} in the bilinear form are defined via norms and mutual angles of some basis vectors e_x , e_y , and e_z

$$\begin{aligned} g_{11} &= \|e_x\|^2 \\ g_{22} &= \|e_y\|^2 \\ g_{33} &= \|e_z\|^2 \\ g_{12} &= \|e_x\| \cdot \|e_y\| \cdot \cos(\angle(e_x, e_y)) \\ g_{13} &= \|e_x\| \cdot \|e_z\| \cdot \cos(\angle(e_x, e_z)) \\ g_{23} &= \|e_y\| \cdot \|e_z\| \cdot \cos(\angle(e_y, e_z)) \end{aligned} \quad (5)$$

Because there is no reason to think that XYZ is an orthonormal system.

In this paper we examine 6 different ways to choose an orthonormal basis in a vector space where vectors represent stimuli. One after another, each inspected system is supposed to be orthonormal. Then length and mutual angles of corresponding XYZ basis vectors are calculated. To determine the relation between XYZ basis vectors (e_x , e_y , e_z), the transition matrix between XYZ and considered coordinate system is used. When coefficients (5) are determined, we can calculate k_{RMS} and k_{\max} values. Results are presented in Table 1.

Table 1. The relation between XYZ basis vectors and coefficients of local non-uniformity for different linear metrics.

Metric	$\ e_x\ /\ e_y\ $	$\ e_z\ /\ e_y\ $	$\angle(e_x e_y)$	$\angle(e_x e_z)$	$\angle(e_y e_z)$	k_{RMS}	k_{max}
Cone	0.336	0.339	117°	101°	85°	15.4	38.5
XYZ	1.0	1.0	90°	90°	90°	11.9	28.6
Lin sRGB	1.390	0.481	146°	114°	81°	6.4	17.7
Cohen	1.026	0.445	142°	106°	82°	5.6	12.3
Photon	0.994	0.481	143°	103°	83°	5.5	11.7
Min_kRMS	0.958	0.465	149°	96°	105°	4.3	8.5

Cone

Suppose Cone Sensitivity Tristimulus values L, M and S are coordinates in an orthonormal system. Color difference in this system should be calculated as follows:

$$D_{Cone}^2 = \Delta L^2 + \Delta M^2 + \Delta S^2 \quad (6)$$

To determine relation between vectors e_x , e_y , and e_z (i.e. their norms and mutual angles) we use matrix transition from XYZ to LMS represented by Stockman, MacLeod & Johnson [2]:

$$\{a_{mn}\} = \begin{pmatrix} 0.236157 & 0.826427 & 0.045710 \\ -0.431117 & 1.206922 & 0.090020 \\ 0.040557 & 0.019683 & 0.486195 \end{pmatrix}$$

Use (6) looks like the most natural choice. However, as it could be seen from the Table1, this choice is far from being optimal.

XYZ

Suppose basic colorimetric coordinate system XYZ is an orthonormal system. Then color difference should be calculated as follows:

$$D_{XYZ}^2 = \Delta X^2 + \Delta Y^2 + \Delta Z^2 \quad (7)$$

Lin sRGB

Suppose linearized basic computer coordinate system Lin sRGB is an orthonormal system. Then color difference in it should be calculated as follows:

$$D_{Lin\ sRGB}^2 = \Delta R_{lin}^2 + \Delta G_{lin}^2 + \Delta B_{lin}^2 \quad (8)$$

To determine relation between vectors e_x , e_y , and e_z we use matrix transition from XYZ to Lin sRGB described in sRGB Standard. Obtained k_{RMS} and k_{max} values correspond to Holm, Tastl, Hordley [4] findings and much smaller, than when we supposed XYZ to be orthonormal.

Cohen

In his doctoral thesis Wyszecki (1953) suggests to represent SPD (Spectral Power Distribution) of any stimulus as a sum of two functions:

$$P(\lambda) = P_f(\lambda) + P_r(\lambda) \quad (9)$$

where $P_f(\lambda)$ is a fundamental component (the main characteristic of color stimulus), which is the same for all stimuli that have an identical influence on a human (its Tristimulus values are equal to

Tristimulus values of $P(\lambda)$), and $P_r(\lambda)$ is a residual or metameric black component, which has no influence upon a human (its Tristimulus values are equal to zero). The choice of the fundamental component involves some arbitrariness.

The arbitrariness might be reduced by fixing three functions $x^*(\lambda)$, $y^*(\lambda)$, $z^*(\lambda)$, whose XYZ Tristimulus values are equal to (1,0,0), (0,1,0) and (0,0,1) accordingly, and request the fundamental component of any stimulus SPD to be equal to a linear combination of these three functions with coefficients X, Y, Z, the Tristimulus values corresponding to this SPD.

$$P_f(\lambda) = X \cdot x^*(\lambda) + Y \cdot y^*(\lambda) + Z \cdot z^*(\lambda) \quad (10)$$

Functions $x^*(\lambda)$, $y^*(\lambda)$, $z^*(\lambda)$ should be called the basis functions (or according to Cohen notation, “fundamental of primaries”).

From mathematical prospective, this method of calculation of $P_f(\lambda)$ is equivalent to calculation of an orthogonal projection of infinite dimensional vector $P(\lambda)$ onto a three dimensional subspace spanned by the basis vectors $x^*(\lambda)$, $y^*(\lambda)$, and $z^*(\lambda)$. Similar sets of basis vectors can be defined for any CMFs set. Moreover, if they are chosen in the same subspace, the value $P_f(\lambda)$ will not be affected.

While the condition (10) reduces the arbitrariness of the fundamental component choice, it does not eliminate it. The arbitrariness can be completely eliminated by developing a rule which determines the choice of the subspace onto which the $P(\lambda)$ is projected.

Cohen [3] created his fundamental color space using the following restriction: fundamental component must be a linear combination of color matching functions.

$$\begin{aligned} x^*(\lambda) &= b_{11}x(\lambda) + b_{12}y(\lambda) + b_{13}z(\lambda) \\ y^*(\lambda) &= b_{21}x(\lambda) + b_{22}y(\lambda) + b_{23}z(\lambda) \\ z^*(\lambda) &= b_{31}x(\lambda) + b_{32}y(\lambda) + b_{33}z(\lambda) \end{aligned} \quad (11)$$

The condition for the Tristimulus values of these functions being equal to (1,0,0), (0,1,0) and (0,0,1) respectively might be written as follows:

$$\begin{aligned} C_n \cdot \int x^*(\lambda)x(\lambda) d\lambda &= 1 & C_n \cdot \int x^*(\lambda)y(\lambda) d\lambda &= 0 & C_n \cdot \int x^*(\lambda)z(\lambda) d\lambda &= 0 \\ C_n \cdot \int y^*(\lambda)x(\lambda) d\lambda &= 0 & C_n \cdot \int y^*(\lambda)y(\lambda) d\lambda &= 1 & C_n \cdot \int y^*(\lambda)z(\lambda) d\lambda &= 0 \\ C_n \cdot \int z^*(\lambda)x(\lambda) d\lambda &= 0 & C_n \cdot \int z^*(\lambda)y(\lambda) d\lambda &= 0 & C_n \cdot \int z^*(\lambda)z(\lambda) d\lambda &= 1 \end{aligned}$$

where C_n is a constant which depends on a choice of a unit of measurement.

This is a system of 9 equations with 9 unknown variables. The solution of the system provides matrix $\{b_{mn}\}$ which determines the relation between $x(\lambda)$, $y(\lambda)$, $z(\lambda)$ and $x^*(\lambda)$, $y^*(\lambda)$, $z^*(\lambda)$ functions.

$$\{b_{mn}\} = \begin{pmatrix} 3.418 & -2.635 & -0.399 \\ -2.635 & 3.247 & 0.204 \\ -0.399 & 0.204 & 0.604 \end{pmatrix}$$

Figure 1 represents $x^*(\lambda)$, $y^*(\lambda)$, and $z^*(\lambda)$ functions for 10° xyz CIE(1964) with Cohen metric.

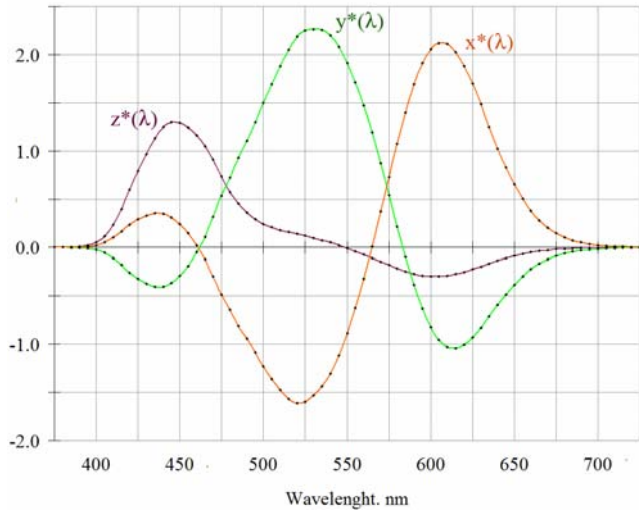


Fig.1. Basis functions for 10° xyz CIE(1964). Cohen metric.

To determine quantitative correlation between basis vectors $x^*(\lambda)$, $y^*(\lambda)$, and $z^*(\lambda)$, a metric must be introduced.

When functions act as vectors (in our case these functions are SPDs), a distance between them is usually computed with the following formula:

$$D^2 = C_{SPD} \cdot \int (P(\lambda) - Q(\lambda))^2 d\lambda$$

where C_{SPD} is a constant which depends on a choice of a unit of measurement. But color difference is defined as follows:

$$D_{Cohen}^2 = C_{SPD} \cdot \int (P_f(\lambda) - Q_f(\lambda))^2 d\lambda \quad (12)$$

This way of the distance computation is equivalent to defining a dot product of two stimuli $P(\lambda)$ and $Q(\lambda)$ as follows:

$$P \bullet Q = C_{SPD} \cdot \int P(\lambda) \cdot Q(\lambda) d\lambda$$

With a usual definition of a norm of a function:

$$\|P\| = \sqrt{P(x) \bullet P(x)}$$

and of an angle between two functions:

$$\cos(\angle(P, Q)) = \frac{P(x) \bullet Q(x)}{\|P(x)\| \cdot \|Q(x)\|}$$

the ratio of norms of basis vectors $x^*(\lambda)$, $y^*(\lambda)$, $z^*(\lambda)$ and their mutual angles might be derived:

$$\|x^*\| : \|y^*\| : \|z^*\| = 1.026 : 1 : 0.445$$

$$\angle(x^*, y^*) = 142^\circ \quad \angle(x^*, z^*) = 106^\circ \quad \angle(y^*, z^*) = 82^\circ$$

The coefficients of local non-uniformity for this coordinate system are as follows:

$$k_{RMS} = 5.6$$

$$k_{max} = 12.3$$

Which are much better, than those provided by traditional metrics.

Photon

One reason why Cohen metric does not become prevalent is the difference in outcome depending on whether wavelength or frequency was used as integration parameter. To eliminate the problem we suggest using weight function $g(\lambda) = \lambda$.

If we substitute the wavelength λ with the frequency ν in the formulas for Tristimulus values calculation, the formulas are transformed as follows

$$\begin{aligned} X &= \int P(\lambda) \cdot x(\lambda) d\lambda = \int P_\nu(\nu) \cdot x_\nu(\nu) d\nu \\ Y &= \int P(\lambda) \cdot y(\lambda) d\lambda = \int P_\nu(\nu) \cdot y_\nu(\nu) d\nu \\ Z &= \int P(\lambda) \cdot z(\lambda) d\lambda = \int P_\nu(\nu) \cdot z_\nu(\nu) d\nu \end{aligned} \quad (13)$$

Where:

$$P_\nu(\nu) = P(c/\nu) \cdot c/\nu^2$$

And:

$$x_\nu(\nu) = x(c/\nu)$$

$$y_\nu(\nu) = y(c/\nu)$$

$$z_\nu(\nu) = z(c/\nu)$$

The substitution does not change X, Y and Z values, but, in case of Cohen metric usage, a value of a dot product of stimuli changes, i.e. the substitution could result in a change of color difference ratio.

Introduction of a weight function into a formula of dot product can eliminate this problem. At that, it should be kept in mind that:

The integrals (13) have a specific physical meaning, which excludes usage of weight function. That is why one cannot vary the formula of dot product of vectors with different unit of measurement, that belongs to different multitudes: SPD and SSD (Spectral Sensitivity of Detector).

The dot product of two vectors with the same unit of measurement (both either SPD or SSD) does not have a specific physical

meaning, and, therefore, a weight function might be included in it as an extra factor. Besides, the transition of variable from wavelength λ to frequency ν for functions that belong to multitude SPD is performed differently then corresponding transition for functions that belong to multitude SSD. Thus the weight function in a dot product between SPD vectors should be different from the weight function in a dot product between CMF vectors.

Tristimulus values do not change if for their calculation modernized functions are used:

$$P_{\text{mod}}(\lambda) = P(\lambda)/g(\lambda)$$

$$x_{\text{mod}}(\lambda) = x(\lambda) \cdot g(\lambda)$$

$$y_{\text{mod}}(\lambda) = y(\lambda) \cdot g(\lambda)$$

$$z_{\text{mod}}(\lambda) = z(\lambda) \cdot g(\lambda)$$

$$X = \int P(\lambda) \cdot x(\lambda) \, d\lambda = \int P_{\text{mod}}(\lambda) \cdot x_{\text{mod}}(\lambda) \, d\lambda$$

$$Y = \int P(\lambda) \cdot y(\lambda) \, d\lambda = \int P_{\text{mod}}(\lambda) \cdot y_{\text{mod}}(\lambda) \, d\lambda$$

$$Z = \int P(\lambda) \cdot z(\lambda) \, d\lambda = \int P_{\text{mod}}(\lambda) \cdot z_{\text{mod}}(\lambda) \, d\lambda$$

Applying the metric without the use of weight functions to modernized functions is equivalent to the use of different weight functions, g^{-2} и g^2 , at defining a dot product of two traditional functions SPDs or CMFs respectively.

$$P \bullet Q = \int P_{f,\text{mod}}(\lambda) \cdot Q_{f,\text{mod}}(\lambda) \, d\lambda = \int P_f(\lambda) \cdot Q_f(\lambda) \cdot g^{-2}(\lambda) \, d\lambda \quad (14)$$

$$x \bullet y = \int x_{\text{mod}}(\lambda) \cdot y_{\text{mod}}(\lambda) \, d\lambda = \int x(\lambda) \cdot y(\lambda) \cdot g^2(\lambda) \, d\lambda \quad (15)$$

Thus the weight function in SPDs dot product should be a reciprocal of weight function in CMFs dot product.

If $g(\lambda) = \lambda^{-1/2}$ (or $g(\nu) = \nu^{1/2}$, respectively) is used as a weight function, then the problem of transition from λ to ν is eliminated because:

$$\begin{aligned} P \bullet Q &= \int P_f(\lambda) \cdot Q_f(\lambda) \cdot g^{-2}(\lambda) \, d\lambda \\ &= \int P_f(\lambda) \cdot Q_f(\lambda) \cdot \lambda \, d\lambda \\ &= \int P_f(c/\nu) \cdot Q_f(c/\nu) \cdot (c/\nu) \cdot c \cdot \nu^{-2} \, d\nu \\ &= \int (P_f(c/\nu) \cdot c/\nu^2) \cdot (Q_f(c/\nu) \cdot c/\nu^2) \cdot \nu \, d\nu \\ &= \int P_{f,\nu}(\nu) \cdot Q_{f,\nu}(\nu) \cdot \nu \, d\nu \\ &= \int P_{f,\nu}(\nu) \cdot Q_{f,\nu}(\nu) \cdot g^{-2}(\nu) \, d\nu \end{aligned}$$

However, metrics with weight function $g(\lambda) = \lambda^{-1/2}$ provides worse results for k_{RMS} and k_{max} , than Cohen metrics.

There is one more dilemma: whether to use SPD or **SPhD** (Spectral Photon Distribution) as a function for stimulus description. The first one is more traditional, but the second one is more appropriate from a prospective of description of interactions of light with detectors.

The transition from SPD to SPhD is equivalent (accurate to a constant) to multiplication of SPD by λ . To eliminate the problem

of transition from λ to ν in this case, the same weight function $g(\lambda) = \lambda^{-1/2}$ should be used. It is easy to see that this expression is equal to the use of weight function $g(\lambda) = \lambda^{1/2}$ (or $g(\nu) = \nu^{1/2}$ respectively) for the dot product of SPDs.

Metrics with weight function $g(\lambda) = \lambda^{1/2}$ provides better results for k_{RMS} and k_{max} parameters, than Cohen metrics. Moreover, this metrics provides the best result among all ones with a weight function $g(\lambda) = \lambda^{n/2}$, where n is a whole number.

So, Photon metrics, determined by the equation:

$$D_{\text{Photon}}^2 = C \cdot \int (P_f(\lambda) - Q_f(\lambda))^2 \lambda^{-1} \, d\lambda \quad (16)$$

corrects some theoretical disadvantages peculiar to Cohen metric and provides a little bit better results for k_{RMS} and k_{max} . However, the difference in value of the parameters is as insignificant, as the difference in basis functions sets which correspond to the same set of CMFs (For example, compare basis functions represented on Fig.1 and Fig. 2).

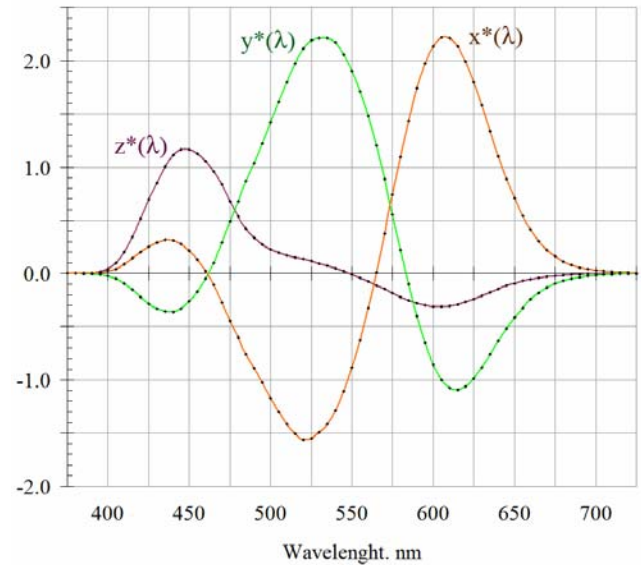


Fig.2. Basis functions for 10° xyz CIE(1964). Photon metric.

Thus, the choice between Photon and Cohen metrics might be important only from theoretical point of view.

Min_ k_{RMS} and an optimal choice of metrics

It looks, like a $\text{Min}_{k_{\text{RMS}}}$ metrics provided by a set of basis vectors which corresponds to the absolute minimum of the function $k_{\text{RMS}}(\|e_x\|/\|e_y\|, \|e_z\|/\|e_y\|, \angle(e_x e_y), \angle(e_x e_z), \angle(e_y e_z))$ might be named as optimal.

However, the criterion of local non-uniformity suggested in the article is only one of several criteria, which has to be considered for optimal color coordinate system. Authors cannot recommend the system named in this article $\text{Min}_{k_{\text{RMS}}}$ as a main color coordinate system. In our opinion, an orthonormal coordinate

system based on Cohen and Wyszecki ideas (*Cohen* or *Photon*) would be a better choice.

Nonlinear Color Coordinate System

Today, an attention of color difference formula designers mainly devoted to nonlinear color coordinate systems. Table 2 represents the parameter of local non-uniformity for some of such systems.

Table 2. Coefficients of local non-uniformity for different nonlinear metrics.

Metric	k_{RMS}	k_{max}
$\Delta E 76$	3.25	6.25
$\Delta E 94$	2.57	4.93
$DE 2000$	2.52	4.49
<i>Cohen 1/3</i>	1.72	2.31

Use of color difference formulas created with heuristic method makes formula comparison much unclear. For example, consider the transaction from *Cohen* metrics to *Cohen 1/3* metrics:

$$D_{Cohen\ 1/3} = (D_{Cohen})^{1/3} \quad (17)$$

As a result of the transaction, k_{max} value decreases from 12.3 to 2.31. This means, that the same rate of local non-uniformity is measured with two different scales using different measuring unit. In this case, there is no reason to interpret the smaller k_{max} value as an evidence of a more uniform space.

Lab system designers create a formula for a^* and b^* calculation which provides zero coordinate values for White (Equal Energy) Stimulus, what makes Lab system more attractive for applications, than XYZ. But there is no clear physical meaning behind that transformation.

In our opinion, Cohen proposes more natural way to get zero coordinates for White Stimulus [3]. He suggests an orthonormal coordinate system with one axis representing a White Stimulus. We incarnate his idea with some modification in our DEF2 coordinate system [5].

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Author Biography

Sergey Bezryadin received MS in physics from the Moscow State University (1976) and PhD in physics and mathematics from the Moscow Institute of Electronic Technique (1982). Since 2001 he has worked in the KWE Int. Inc., San Francisco, USA.